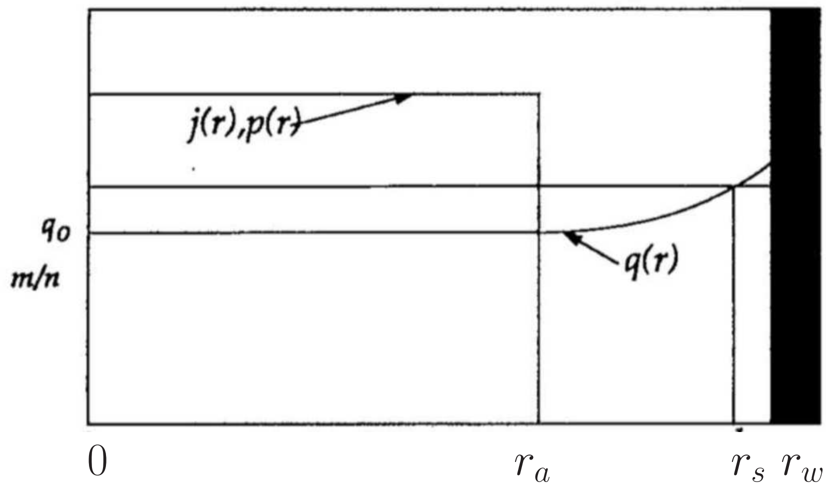

Figure 1: *Finn model equilibrium.*

Resistive Wall Tearing Mode in a periodic cylinder

H. Strauss

The growth rate of a resistive wall tearing mode (RWTM) can be obtained from a limit of the dispersion relation obtained in [2]. The growth rate of a resistive wall tearing mode (RWTM) can be obtained from a limit of the dispersion relation obtained in [2]. The linear growth rate of the tearing mode (TM) [1] is given by

$$\gamma\tau_A = c_1(\Delta' r_s)^{4/5} S^{-3/5} \tag{1}$$

$$c_1 = 0.55\left(\frac{mq' r_s}{q^2}\right)^{2/5} \tag{2}$$

where r_s is the rational surface and m is the poloidal mode number. For simplicity, a zero pressure circular large aspect ratio geometry is assumed. The Finn model [2, 4] assumes a constant current density $j = 2/q_0$ for $r < r_a$. For $r > r_a$ $j = 0$ and $q = q_0(r/r_a)^2$. The rational surface $q = q_s = m/n$ occurs at $r_s/r_a = (q_s/q_0)^{1/2}$. At the wall $r = r_w$ [2]

$$\Delta'_w = \frac{2m}{r_s}\frac{nq_0 - (m-1) - (r_a/r_w)^{2m}}{[nq_0 - (m-1) - (r_a/r_s)^{2m}][1-(r_s/r_w)^{2m}]} \tag{3}$$

if the wall is ideal, then the condition $\Delta'_w = 0$ implies q_0 is

$$nq_0 = (m-1) + (r_a/r_w)^{2m}. \tag{4}$$

For $(m,n) = (2,1)$, then $q_0 = 1 + (r_a/r_w)^4$. Combining,

$$\frac{r_s}{r_a} = \left[\frac{2}{1+(r_a/r_w)^4}\right]^{1/2} \approx \sqrt{2} \tag{5}$$

for $(r_a/r_w)^4 \ll 1$. The no wall Δ' is

$$\Delta'_\infty = -\frac{2m}{r_s}\frac{nq_0 - (m-1)}{nq_0 - (m-1) - (r_a/r_s)^{2m}},$$

which is Δ'_w with $r_w \to \infty$. Using the condition $\Delta'_w = 0$ gives

$$\Delta'_\infty = \frac{2m}{r_s}\frac{1}{(r_w/r_s)^{2m} - 1}. \tag{6}$$

Then [2]

$$\Delta' = \frac{R\Delta'_w + \Delta'_\infty}{1+R}$$

where the quantity R is defined

$$R = \frac{\gamma\tau_{wall}}{2m}[1 - (r_s/r_w)^{2m}]$$

Assuming $R \gg 1$ and $\Delta'_w = 0$ the dispersion relation was obtained in [3],

$$\gamma\tau_A = \frac{c_0}{S^{1/3}S_{wall}^{4/9}} \tag{7}$$

where

$$c_0 = 2.46\left(\frac{q' r_s}{q}\right)^{2/9} f^{4/9} = 2.46 f^{4/9} \tag{8}$$

$$f = \frac{(r_s/r_w)^{2m}}{[1 - (r_s/r_w)^{2m}]^2} \tag{9}$$

The condition $R \gg 1$ for the RWTM is

$$R = \frac{c_0 S_w^{5/9}}{4S^{1/3}}\left[1 - \left(\frac{r_s}{r_w}\right)^4\right] \approx (r_s/r_w)^{8/9}[1-(r_s/r_w)^4]^{1/9} S_w^{5/9} S^{-1/3} \gg 1. \tag{10}$$

For example, let $r_w = 1$, $r_a = 0.6$, then $r_s = 0.798$, and $q_0 = 1.1296$. Then $R \gg 1$ inplies $S_w \gg 3.9 S^{3/5}$. If $S = 10^3$, then $S_w \gg 246$.

If $R \ll 1$, then the dispersion relation is (1) with

$$\Delta' = \Delta'_\infty. \tag{11}$$

The condition $R \ll 1$ for the TM is

$$R = c_1(r_s\Delta'_\infty)^{4/5} S^{-3/5} \ll 1 \tag{12}$$

which is of order $S^{3/5} \gg 1$.

The idea is to solve reduced MHD in a straight cylinder as in [5] and compare with the theory. An important result will be to find the sensitivity of the answers to the resolution in the resistive wall, and see if it depends on τ_{wall}.

$$\frac{\partial \psi}{\partial t} = \mathbf{B} \cdot \nabla \phi + \eta \nabla^2 \psi \tag{13}$$

$$\frac{\partial \nabla^2 \phi}{\partial t} = \mathbf{B} \cdot \nabla \nabla^2 \psi \tag{14}$$

$$\mathbf{B} \cdot \nabla \phi = \nabla \psi \times \nabla \psi \cdot \hat{\zeta} + \frac{R_0}{R} \frac{\partial \phi}{\partial \zeta} \tag{15}$$

To do with M3D-C1 this there are two preliminary steps. First, construct a mesh. The Simmodeler meshes have triangles much more equilateral than in [5], which are more acute and flux surface aligned. The second step is to initialize with a step function current density equilibrium.

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