Pressure coupling

- MHD momentum equation including pressure coupling of energetic ions

$$\rho\left[\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\cdot\nabla)\,\mathbf{V}\right]+\frac{\partial \mathbf{M}_{hot}}{\partial t}=\mathbf{J}\times\mathbf{B}-\nabla p-\nabla\cdot\mathbf{P}_{hot}$$

$$\mathbf{J}=\nabla\times\mathbf{B}$$

- After linearization

$$\rho\frac{\partial \mathbf{V}_1}{\partial t}=\mathbf{J}_0\times\mathbf{B}_1+\mathbf{J}_1\times\mathbf{B}_0-\nabla p_1-\nabla\cdot\mathbf{P}_{hot,1}$$

- $\mathbf{P}_{hot,1}$ is calculated following δ-f method.

Current coupling

- MHD momentum equation including current coupling

$$\rho \left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right] = (\mathbf{J} - \mathbf{J}_{hot}) \times \mathbf{B} - \nabla p$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

- After linearization

$$\rho \frac{\partial \mathbf{V}_1}{\partial t} = \mathbf{J}_0 \times \mathbf{B}_1 + \mathbf{J}_1 \times \mathbf{B}_0 - \mathbf{J}_{hot,0} \times \mathbf{B}_1 - \mathbf{J}_{hot,1} \times \mathbf{B}_0 - \nabla p_1$$

- $\mathbf{J}_{hot,1}$ is calculated following δ-f method. $\mathbf{J}_{hot,0}$ comes from the equilibrium and must be calculated separately.

Calculation of coupling term

$P_{||}$ and $P_{\perp}$ can be used for both pressure and current coupling to MHD equations.

- Pressure coupling

$$\nabla \cdot \mathbf{P} = \nabla P_{\perp} + \nabla \cdot \left(P_{||} - P_{\perp}\right) \mathbf{bb}$$

- Current coupling

$$\mathbf{J}_{hot} \times \mathbf{B} = \frac{P_{||}}{B^2}\mathbf{b} \times \nabla \times \mathbf{b} - \frac{P_{\perp}}{B^2}\nabla_{\perp} \ln B - \nabla \times \left(\frac{P_{\perp}}{B}\mathbf{b}\right) \times \mathbf{B}$$

  - For equilibrium with $P_{||} = P_{\perp} = p_{hot}$

$$\mathbf{J}_{hot,0} \times \mathbf{B}_1 = \mathbf{b}_0 \frac{\mathbf{B}_{1,\perp}}{B_0} \cdot \nabla p_{hot}$$

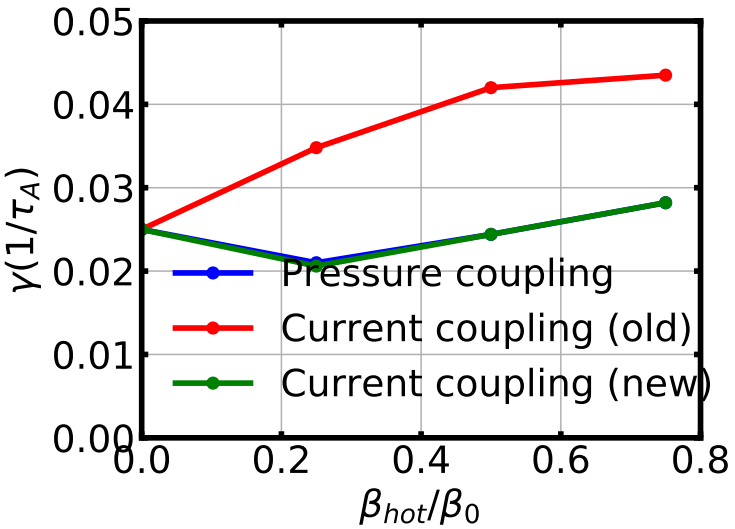
  - Current coupling is consistent with pressure coupling in linear approximation assuming $\mathbf{B}_{1,\perp} \cdot \nabla p_{hot} = \mathbf{B}_0 \cdot \nabla \cdot \mathbf{P}_{hot,1}$

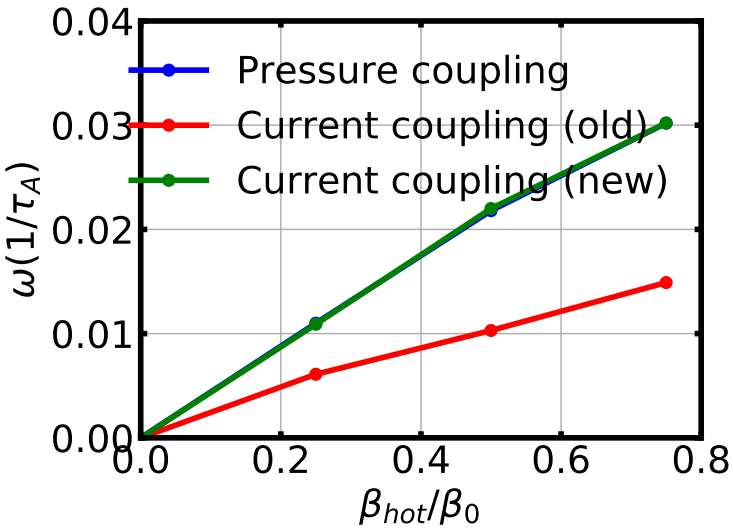
Y. Todo and T. Sato, Phys. Plasmas 5, 1321 (1998).

Linear fishbone simulation using pressure and current coupling

$R/a = 2.8, \quad \beta_{total} = 0.08, \quad q_0 = 0.6, \quad q_a = 2.5$

$\hat{\rho}_h = v_0/(\Omega_h a) = 0.0125, v_0/v_A = 4$

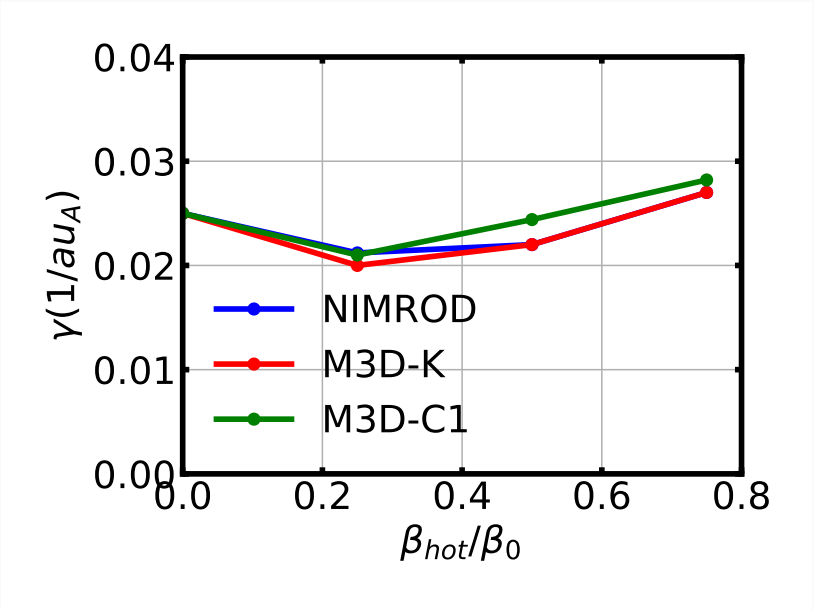


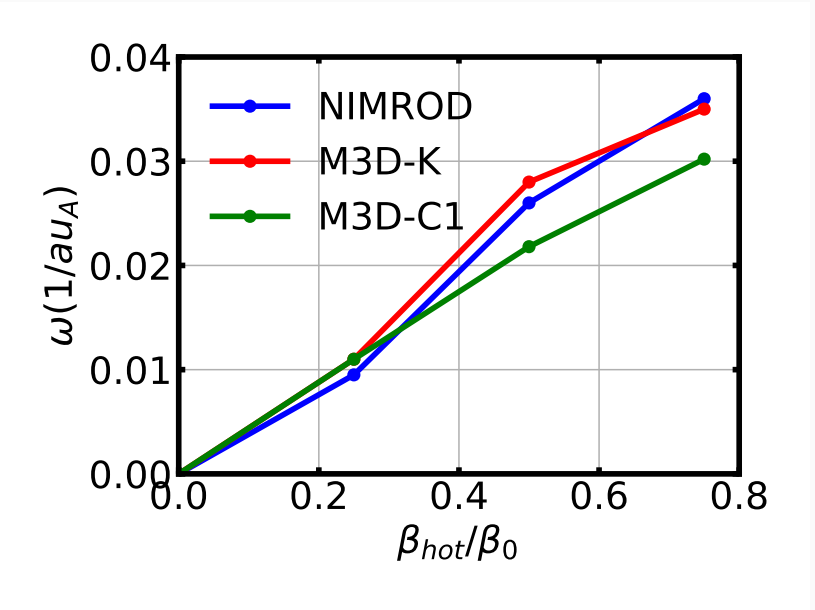


Benchmark with other codes

$R/a = 2.8, \quad \beta_{total} = 0.08, \quad q_0 = 0.6, \quad q_a = 2.5$

$\hat{\rho}_h = v_0/(\Omega_h a) = 0.0125, v_0/v_A = 4$





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C.C. Kim and the NIMROD Team, Phys. Plasmas 15, 072507 (2008).