SuperLU_DIST algorithms flowchart

Nonsymmetric pattern
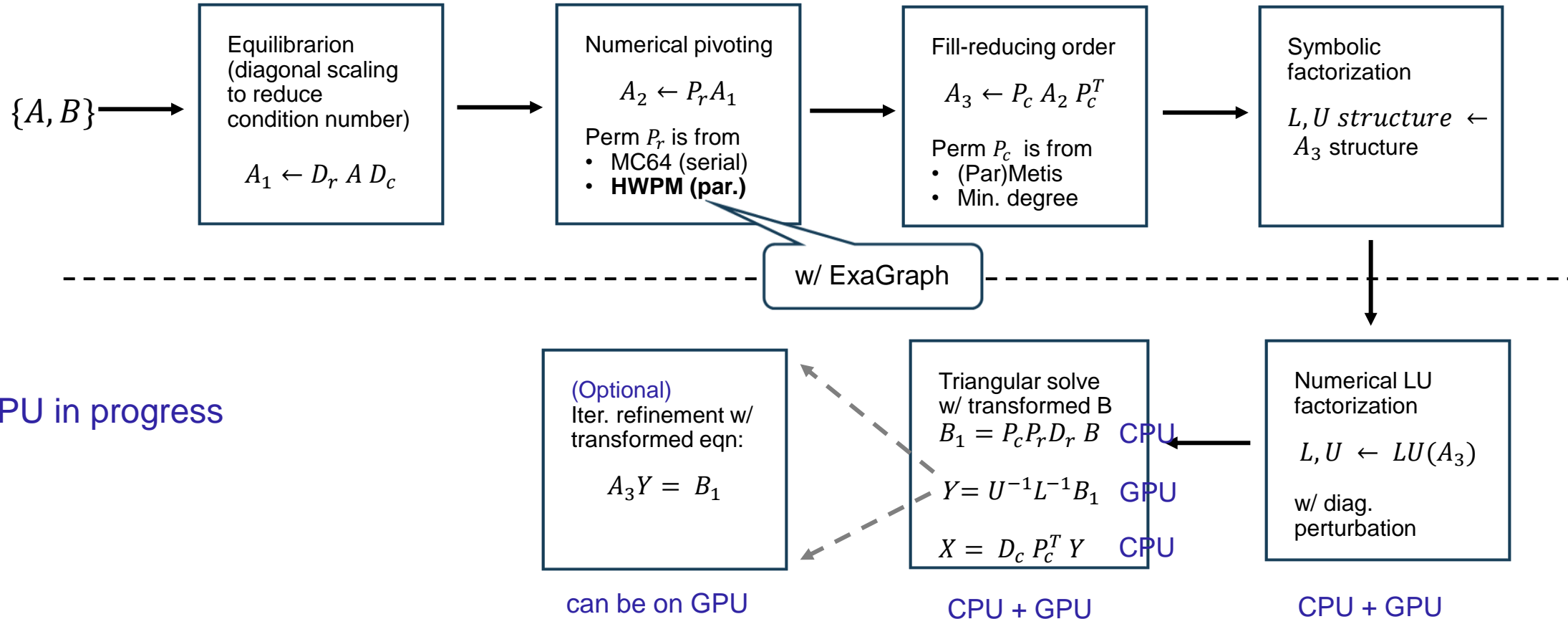
SuperLU

Preprocessing mostly on CPU

GPU easy

GPU hard

GPU hard

GPU in progress

$\{A, B\}$ →

Equilibrarion (diagonal scaling to reduce condition number)

$$A_1 \leftarrow D_r\, A\, D_c$$

Numerical pivoting

$$A_2 \leftarrow P_r A_1$$

Perm P_r is from
- MC64 (serial)
- **HWPM (par.)**

Fill-reducing order

$$A_3 \leftarrow P_c\, A_2\, P_c^T$$

Perm P_c is from
- (Par)Metis
- Min. degree

Symbolic factorization

$$L, U\ structure\ \leftarrow A_3\ structure$$

w/ ExaGraph

GPU in progress

(Optional) Iter. refinement w/ transformed eqn:

$$A_3 Y = B_1$$

Triangular solve w/ transformed B
$$B_1 = P_c P_r D_r\, B \quad \text{CPU}$$
$$Y = U^{-1} L^{-1} B_1 \quad \text{GPU}$$
$$X = D_c\, P_c^T\, Y \quad \text{CPU}$$

Numerical LU factorization

$$L, U\ \leftarrow\ LU(A_3)$$

w/ diag. perturbation

can be on GPU

CPU + GPU

CPU + GPU

Solve the transformed system: $(P_c\, P_r\, D_r\, A\, D_c\, P_c^T)(P_c\, D_c^{-1}\, X) = P_c\, P_r\, D_r\, B$