1D Runaway Electron Formation

$$\dot{\psi} = \frac{\eta}{\mu_0}\left[\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \psi}{\partial r} - \mu_0 R J_{RE}\right] \qquad y = r^2 \quad \dot{\psi} = \frac{\eta}{\mu_0}\left[4\frac{\partial}{\partial y} y \frac{\partial \psi}{\partial y} - \mu_0 R J_{RE}\right]$$

$$\frac{dn_r}{dt} = n_e \nu_{ee} E^{-3(1+Z)/16} \exp\left[-1/(4E) - \sqrt{(1+Z)/E}\right] \quad \nu_{ee} = n_e e^4 \ln\Lambda / 4\pi\varepsilon_0^2 m_e^2 v_{th}^3 \qquad v_{th} = \sqrt{2T_e / m_e}$$

$$E = (T_e \ / m_e c^2)(E_{EF} \ / E_c) \quad E_c = n_e e^3 \ln\Lambda / 4\pi\varepsilon_0^2 m_e c^2 \quad E_{EF} = \frac{\eta}{\mu_0}\left[\frac{4}{R}\frac{\partial}{\partial y} y \frac{\partial \psi}{\partial y} - \mu_0 J_{RE}\right]$$

$$T_e = 10^3 eV \qquad n_e = 10^{20} m^{-3}$$

$$v_{th} = \left[\frac{T_e \times 2 \times 1.1604 \times 10^4 K \times 1.3807 \times 10^{-23} J/K}{9.1094 \times 10^{-31} kg}\right]^{1/2} = 5.93 \times 10^5 \sqrt{T_e} ms^{-1}$$

$$\nu_{ee} = \frac{n_0 m^{-3} \times \left(1.6022 \times 10^{-19} C\right)^4 \times 20}{4\pi\left(8.8542 \times 10^{-12} Fm^{-1}\right)^2 \left(9.1094 \times 10^{-31} kg\right)^2 \left(5.93 \times 10^5 \sqrt{T_e} ms^{-1}\right)^3} = 7.731 \times 10^{-11} n_0 T_e^{-3/2} s^{-1}$$

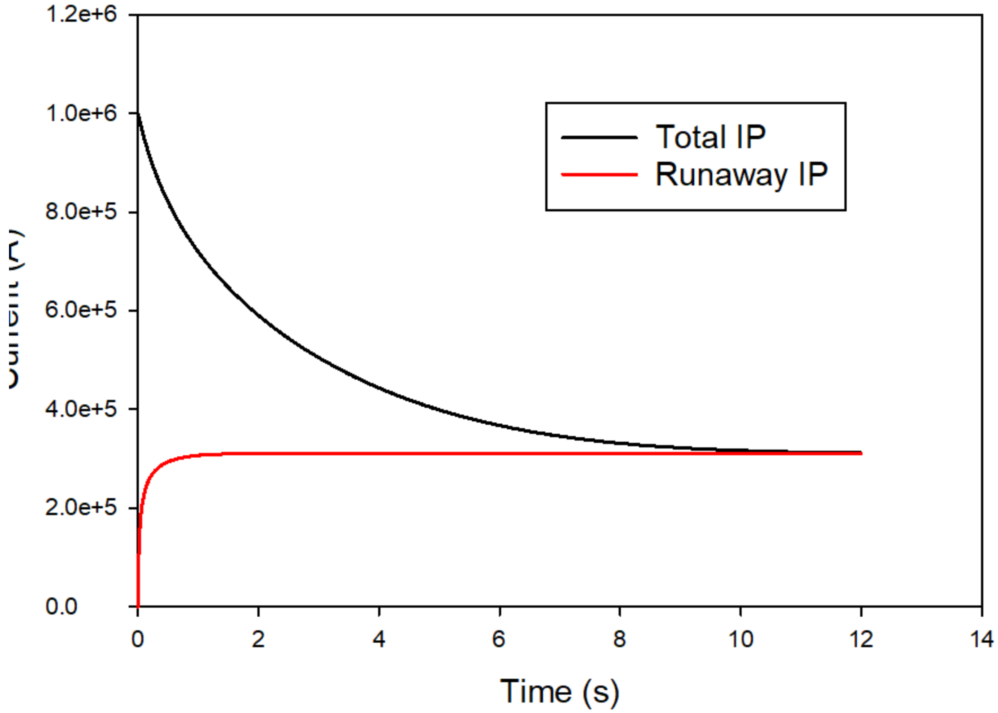
$$E_c = \frac{n_0 m^{-3} \times \left(1.6022 \times 10^{-19} C\right)^3 \times 20}{4\pi\left(8.8542 \times 10^{-12} Fm^{-1}\right)^2 \left(9.1094 \times 10^{-31} kg\right)\left(3.0 \times 10^8 ms^{-1}\right)^2} = 1.018 \times 10^{-21} n_0 \ Vm^{-1}$$

$$E = \frac{T_e \times 1.1604 \times 10^4 K \times 1.3807 \times 10^{-23} J/K}{\left(9.1094 \times 10^{-31} kg\right)\left(3.0 \times 10^8 ms^{-1}\right)^2}(E_{EF} \ / E_c) = 1.95 \times 10^{-6} T_e (E_{EF} \ / E_c)$$

$$\eta = 1.03 \times 10^{-4} \times 20 \times T_e^{-3/2} \Omega m = 2.06 \times 10^{-3} T_e^{-3/2} \Omega m \quad \eta / \mu_0 = \frac{2.06 \times 10^{-3} T_e^{-3/2}}{4\pi \times 10^{-7}} = 1.639 \times 10^3 T_e^{-3/2} m^2 s^{-1}$$

Initial Condition

$$\frac{\partial}{\partial y} y \frac{\partial \psi}{\partial y} = \tfrac{1}{4} R J_0 \left[1 - \left(\frac{y}{a^2} \right)^2 \right], \qquad \psi = \tfrac{1}{4} R J_0 \left[y - \frac{y^3}{9a^4} \right]$$



Linearize the source function about the present time

$$n_r^{n+1} = n_r^n + dt\, n_e \nu_{ee} f(E^{n+1})$$

$$= n_r^n + dt\, n_e \nu_{ee} \left[f(E^n) + f'\left(E^{n+1} - E^n\right) \right]$$

$$f(E) = E^{-3(1+Z)/16} \exp\left[-1/(4E) - \sqrt{(1+Z)/E} \right]$$

$$f' = \left(-3(1+Z)/16\right) E^{-3(1+Z)/16-1} \exp\left[-1/(4E) - \sqrt{(1+Z)/E} \right]$$

$$+ E^{-3(1+Z)/16} \exp\left[-1/(4E) - \sqrt{(1+Z)/E} \right] \left[1/4E^2 + \sqrt{(1+Z)}/2E^{3/2} \right]$$

$$E^{n+1} = (T_e / m_e c^2)(1/E_c) \frac{\eta}{\mu_0} \left[\frac{4}{R} \frac{\partial}{\partial y} y \frac{\partial \psi^{n+1}}{\partial y} - \mu_0 e c n_r^{n+1} \right]$$

$$= G \left[\frac{4}{R} \frac{\partial}{\partial y} y \frac{\partial \psi^{n+1}}{\partial y} - \mu_0 e c n_r^{n+1} \right] \qquad G \equiv (T_e / m_e c^2)(1/E_c) \frac{\eta}{\mu_0}$$

Consider the joint implicit method

$$n_r^{n+1} = n_r^n + dt\, n_e \nu_{ee} \left[f(E^n) + f' \left(G \left[\frac{4}{R} \frac{\partial}{\partial y} y \frac{\partial \psi^{n+1}}{\partial y} - \mu_0 e c n_r^{n+1} \right] - E^n \right) \right]$$

$$\psi^{n+1} = \psi^n + dt \frac{\eta}{\mu_0} \left[4 \frac{\partial}{\partial y} y \frac{\partial \psi^{n+1}}{\partial y} - \mu_0 \,\mathrm{Re}\, c n_r^{n+1} \right]$$